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The Birth of Logic

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The Birth of Logic

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Contents

Abstract

1. Demonstrative Proofs: The Subject-Matter of Prior Analytics
2. The Aristotelian Revolution: A Radical Shift of Focus
3. Aristotle's Theory of Deduction: Direct and Indirect Deductions

Abstract: The last two decades have witnessed a debate concerning whether Aristotle's syllogistic is a system of deductive discourses having epistemic import exemplifying an Aristotelian theory of deductive reasoning and justifying the claim that Aristotle is the founder of logic taken as the scientific study of proof or whether, on the contrary, the syllogistic is a system of true propositions of a theory of classes justifying the claim that Aristotle is the founder of logic is taken as the scientific study of formal relations such as class inclusion. An epistemically-oriented interpretation has been contending with an ontically-oriented interpretation. This debate should not be confused with the related issue, which is partly terminological, of whether logic should be construed as an organon and epistemic metascience of reasoning or as an ontic science on a par with but antecedent to, and more abstract than, other sciences. The present nontechnical, nonpolemical, expository essay attempts to show that approaching Aristotle's logical writings from a standpoint informed by knowledge and appreciation of the scientific and philosophical achievements of Aristotle's predecessors, especially Socrates, Plato and the Academic mathematicians, (rather than from the standpoint of the logicistic, Frege-Russell paradigm) will make the epistemically-oriented interpretation more plausible than the ontically-oriented one. The epistemically-oriented interpretation permits the birth of logic as epistemic metascience to be located with Aristotle while deferring the birth of logic as ontic science to the modern period. In contrast, the ontically-oriented interpretation permits the birth of logic as ontic science to be located with Aristotle while deferring the birth of logic as epistemic metascience to the modern period.

1. Demonstrative Proofs: The Subject-Matter of Prior Analytics. Aristotle himself tells us in Prior Analytics that his subject is apodeixis (demonstration, proof, or demonstrative proof). Historians of mathematics date the origins of the practice of demonstrative proof centuries before Aristotle. Predating Aristotle by decades was one of his favorite examples, a proof that the side of the square and its diagonal cannot both be measured in a whole number of units of a fixed length. This remarkable result concerning ideal (or abstract) geometrical figures has no meaningful analogue in the practical experience of engineers, surveyors, and carpenters. Given any material square, a sufficiently small unit can be found which will measure in whole numbers of lengths both the side and the diagonal--within the limits of experimental accuracy.

One ancient proof of this theorem uses as a premise the proposition now known as the Pythagorean Theorem, which had itself been proved several decades earlier yet. The theorem in question, that the side of the square is incommensurable with the diagonal, is a theorem of geometry about square figures but it is closely related to a theorem of arithmetic about square numbers, viz. the proposition that no square number is double of another square number, in other words that no two square numbers, no matter how large, are in the ratio of one-to-two.

Pairs of square numbers can be found in a ratio closer to the ratio of one-to-two than any given ratio, no matter how close. We can get pairs of square numbers as close as we want to the chosen ratio but we can never reach it. The practical experience of continual frustration in trying to find a square whose double is also a square is reflected in and predictable from this theorem, which has received attention from many philosophers including Pascal, Descartes and Leibniz.

The Double Square Number Theorem, if I may call it that, was of course known to Plato, as we learn from the Theatetus. It is in sharp contrast to a geometrical theorem which also must have certainly been known to Plato. I refer here to the proposition that every square figure is double of a square figure, a result easily deducible from results explicitly mentioned in the Meno. The Double Square Number Theorem, that no square number is double of a square number, contrasts with the Double Square Figure Theorem, that every square figure is double of a square figure.

I hope that you will excuse me for reminding you of these elementary but representative facts about the state of the art of demonstration before Aristotle. The interpretation that we put on a work is colored by what we know, by what we have in mind when we read it, and by what we could imagine ourselves to be concerned with were we to change places with the author. I do not see how it could be possible to understand the Analytics without having the experience of knowing geometrical and arithmetic theorems, without being struck (perhaps stunned would be a better word) by the cogency of demonstrative proof, and without some awareness of the historical situation involving demonstrative proof. Interpretations of Aristotle's syllogistic not informed by these prerequisites seem to find Aristotle's syllogistic to be alienating, formalistic, simplistic, narrow, useless, labored, and uninspiring and they seem to suffer the same faults themselves.

In order to understand what study Aristotle was proposing to undertake it is necessary to have some familiarity with the subject-matter of that study as

it had become manifest at that time to Aristotle and to Aristotle's contemporaries, the persons to whom Aristotle addressed the Prior Analytics. What a writer chooses to say about a subject-matter depends to some extent on what that writer thinks the reader already knows or believes.

Euclid's Elements can provide us with a good impression of what examples of proofs were available to Aristotle's readership. Unfortunately, the Elements were written after Prior Analytics and it is therefore possible that its style of presentation of proof was influenced by the theory of proof found in Prior Analytics. Nevertheless, the propositions that are proved in the Elements had for the most part already been proved before Aristotle undertook his study of proof and, as far as we know, the proofs that had been given before Aristotle's study resemble in essential ways those in Euclid.

Not only had most of the propositions proved in Euclid's Elements been proved before Aristotle's time, it is also the case that there were axiomatizations of geometry already available in the Academy. It is altogether possible that a project of axiomatizing geometry was underway in the Academy while Aristotle was there. Be that as it may, there was a considerable body of proofs available to Aristotle as data for his study.

The function of a proof, of course, is production of knowledge. Every proposition which is proved to a given person is known to be true by that person. This function of proof is what makes demonstrative science possible. Proof has the remarkable capacity to bring about unwavering and stable belief in cases where we would otherwise be condemned to suspended judgement or at best to an attenuated moral certainty; proof makes it possible for us to form responsible belief when otherwise belief, if achieved at all, would be irresponsible. How could you decide whether every number which is twice a square is non-square? By trying out the first few squares, 1, 4, 9, 16, 25, we merely illustrate to ourselves the experimental import of the proposition. In some cases an expectation may be engendered, but this expectation becomes hedged once we realize that it could be upset by one counterexample.

The capacity for a proof to establish belief is closely related to its capacity to extinguish doubt. Certain propositions seem to have the curious capacity to raise doubts and to create tension. For example, people that are not used to the following proposition tend to doubt it when they encounter it: every triangle having the square on one of its sides equal in area to the sum of the squares on the other two sides is right-angled. This, of course, is the converse of a corollary to the Pythagorean Theorem and it is proved in Euclid immediately after the Pythagorean Theorem. The Pythagorean Theorem enables us to infer something about the sides of a triangle given information about its angles; the Converse Pythagorean Corollary enables us to infer something about the angles given information about the sides.

2. The Aristotelian Revolution: A Radical Shift of Focus. The practice of demonstrative proof evidently first took root in Ionia during or shortly before the time of Thales, perhaps a little more than two centuries before Aristotle undertook its study. The taking root of this practice was followed by a flowering of learning, both scientific and humanistic, and it is impossible to conceive of the scientific development without demonstrative proof. How the practice came about we do not know. We do not know whether it was sudden or gradual. We do not know whether it was largely due to one or a small number of geniuses or whether it was essentially a communal development. Shifting our

attention from the historical origin of the practice to the acquisition of it by individuals today leaves us just as short of answers.

Learning to follow demonstrative proofs is a tricky business. Despite an almost total absence of insight into how it is done, many of us manage to do it. Discovering new demonstrative proofs is another remarkable skill that many of us learn to do quite well.

Little has been written on these two epistemic skills despite the fact that discovering new proofs is an activity that can yield deep inner satisfaction, feelings of accomplishment and self-worth, feelings of competence and feelings of community with other human beings--not to mention the aesthetic enjoyment involved. Following a demonstrative proof has similar benefits which, I might add, seem to be intimately related to the acquisition of knowledge. Others have pointed out these things as well as the further point that following a proof feels a lot like discovering a proof but being helped along by hints. The gratifications achieved in the course of following and creating proofs rank high among the joys of the intellectual life. There is a special happiness that accompanies the search for a proof when the search lasts a long time, say months or years. It has been referred to as a feeling of being pregnant with a scientific child. My own suspicion is that the impressions of freshness, hope, liveliness and dignity that we get from our study of ancient Greek thought is due in part to the benefits of demonstrative proof.

In the course of creating or absorbing a demonstration one's attention is focused on the subject-matter, be it geometrical, arithmetic, set-theoretic, or what-not. Not only is the language in use transparent, to use Polanyi's apt phrase, but also out of focus and maybe out of the field of vision altogether are the propositions and inferential connections involved in the demonstration. Plato, Aristotle and Proclus have made scattered remarks about the demonstrative or apodictic experience. I should like to emphasize that I am using the word 'experience' as it is used in normal English and not as it is used by the so-called empirically-oriented philosophers who limit the term to sensation. The point that I am getting to about the activity of creating or absorbing an actual demonstration is this: in the course of this activity the mind is intensely focused, perhaps riveted, on the subject-matter. Try it out. The Meno was written by someone who knows what it is to do a demonstration.

An early step in the founding of a science of proof involves shifting one's focus from the subject-matter to the process of thought involved in the demonstration. When a person first starts to attempt to study demonstration itself there are many vertiginous moments, there are moments when one loses one's grip on one's thoughts, it is something like trying to observe the series of mouth positions involved in pronouncing a word, or perhaps trying to observe the spectrum of leg-positions involved in jumping a hurdle. These analogies are not quite apt. Another analogy that comes to mind is studying the motion of a tool in a process instead of focusing on the task, for example in the process of sweeping the sidewalk instead of attending to the debris being swept attend instead to the motion of the broom.

There are several other steps involved in the process of moving from a study of a given subject-matter, say geometry, to a study of proofs about that subject-matter and, more generally, to a study of proofs themselves. I call this event or process the Aristotelian Revolution.

Another step is the discovery of abstract, timeless, static, entities underlying the individual concrete, temporal, dynamic, processes of demonstrative proof. When we experience proofs we may be said to be involved in a proof-performance or proof-token as opposed to an abstract proof or proof-type. An abstract proof is an allographic artifact like a poem or a song which admits of individual, concrete, temporal performances; allographic artifacts are contrasted with the so-called autographic artifacts such as paintings. Historians of mathematics, including Morris Kline, have credited the Greeks with making mathematical science possible by discovering how to treat numbers and geometrical figures as abstractions related to but apart from concrete multitudes and concrete shaped-things. The analogous step in regard to proof may safely be credited to Aristotle; in any case the Prior Analytics is the earliest known work which treats proofs as timeless abstractions amenable to investigation similar to the investigations already directed toward numbers and geometrical figures. These points and other very closely related points have been made by other writers including Robin Smith and James Gasser. A person who has read Plato and who has experienced first-hand some of the mathematics done in the Academy is not likely to read the Prior Analytics without being struck by Aristotle's hypostasization of proof, provided of course that the person is alert to the issue.

It was already obvious before Aristotle that a single proof typically involved several concepts (or terms), that besides the conclusion (or proposition being proved) there were premises whose truth needed to be established before the proof could be made, that there are still further propositions besides the premises and the conclusion, and that there are two contrasting kinds of proof; on one hand we have direct proofs such as Euclid's proof of the Pythagorean Theorem which so-to-speak builds up to the conclusion and on the other we have indirect proofs such as the usual proof of the Square Incommensurability Theorem which so-to-speak derives an impossibility from the supposition of the opposite of the conclusion. But some things that seemed to be obvious were found by Aristotle to be false. For example, examination of the proofs in Euclid, say, gives the distinct impression that many if not all proofs involve manipulation of the entities that the conclusions are about; proofs in geometry seem to involve manipulation of geometrical figures and proofs in arithmetic seem to involve the manipulation of numbers. Aristotle makes the point himself. Even today discussion of mathematical proof is pervaded by constructional language, sometimes avowedly metaphorical, but sometime avowedly literal. Aristotle's theory of proof has no room for manipulation of subject-matter. For Aristotle the only epistemic activities involved in proof, once the premises are secured, are inferring (or applying rules of inference), assuming (or making suppositions), and more or less clerical activities such as remembering and recognizing.

Many later thinkers including Kant accepted the existence of reasoning in accord with Aristotle's theory but they could not accept the view that Aristotle's theory was exhaustive. Put another way, they accepted Aristotelian reasoning but they considered it only a species of a wider genus which also includes "synthetic" or "constructional" reasoning; in particular geometrical reasoning and arithmetical reasoning were thought of as involving constructions alien to Aristotle's purely inferential, nonmanipulational view. Even to this day intuitionistic followers of Brouwer hold that arithmetic proof involves constructions.

According to some logicians, including Beth, the most important discovery

by Aristotle was the idea that proof consists in inference of consequences of premises known to be true. This has been called the Truth-And-Consequence Conception of proof. This view recognizes that the inferential aspect of proof, the deducing of logical consequences from premises, is separable from the material aspect, the epistemic apprehension of the truth of the premises. It allows for the universalization of the inferential aspect, the idea that inference is one and the same regardless of subject-matter however much the material aspect may vary from one subject-matter to another. In other words, it opens the door to formal logic.

It is the truth-and-consequence conception of proof that underlies Aristotle's distinction between demonstrations and deductions, i.e. between apodeixis and sullogismos. A deduction makes evident that its conclusion follows logically from its premise-set. A proof is a deduction whose premises are known to be true. Plato may have something like this distinction in the Republic when he points out that knowledge of the theorems of geometry depends on knowledge of the basic hypotheses.

As Aristotle himself tells us, every demonstration is a deduction but not every deduction is a demonstration. In Prior Analytics Aristotle discusses deductions in general much more than just proofs per se; although his goal is understanding of proof, the species, he finds that the goal requires understanding of the genus, deduction.

The word argumentation has been used to refer to the genus that has the class of deductions as a species. Every deduction is an argumentation and but not every argumentation is a deduction. An argumentation may be thought of as composed of a premise-set, a conclusion and a discourse (or chain of reasoning) which may or may not be fallacious. It has been pointed out by other writers that the word argumentation is a convenient translation for logos in some but not all of its occurrences in Plato and Aristotle.

The entity that we refer to as the Euclidean proof of the Pythagorean Theorem is clearly an argumentation. Its premises include several axioms and definitions and perhaps also some previously established propositions. Its conclusion is the Pythagorean Theorem, the proposition that in every right triangle the square on the hypotenuse is equal in area to the combination of the squares on the other two sides. The discourse or chain of reasoning is described by a relatively long text which is informally said to include among other things an explanation of how to cut the square on the hypotenuse into two rectangles which may be shown to equal respectively the other two squares. In typical cases the chain of reasoning strikes us as being much longer than the combination of the premises and the conclusion and moreover the chain of reasoning has a semantic character radically different from that of a set of propositions or a single proposition. Propositions have a semantically static character in that they do not explain or report a process of reasoning. A chain of reasoning on the other hand may be said to be a recipe for carrying out a mental process. To use Austin's terminology, the premises and conclusion of a demonstration are expressed by declaratives whereas the chain of reasoning must be expressed by a performative. In order to follow a proof it is necessary to carry out the processes set forth in the chain of reasoning. Reading a proof-text with understanding, requires reader participation to a much greater extent and in much more striking way than does reading a story. Reading a proof-text is not a spectator activity. A proof makes a prediction that the reader must verify in order to understand the proof.

3. Aristotle's Theory of Deduction: Direct and Indirect Deductions. The word argument has several meanings in normal English but in logic it has a technical meaning that it rarely takes elsewhere. In logical discussions an argument is a two-part system composed of a set of propositions called its premises and a single proposition called its conclusion. There are two very elementary points to be made: an argument is not a molecular proposition because its constituent propositions are not combined by connectives into one, a typical proof contains a non-propositional discourse or chain of reasoning (and thus is not an argument). Every proof contains an argument in the sense that it has a premise-set and a conclusion but it is not itself an argument per se. The difference between containing an argument and being an argument is important.

Mates has alleged that the Stoics thought that demonstrations were arguments in this exact sense. Mates also alleged that the Stoics used the word logos as a technical term in logic having the exact sense that we have attached above to argument. If your geometry teacher asked you to do a demonstration of the Pythagorean Theorem and you turned in a set of geometric propositions with the Pythagorean Theorem adjoined, you should expect an exasperated teacher. The teacher wants an argumentation which includes a chain of reasoning, not just an argument, which per se necessarily lacks any inferential recipe.

Just as propositions divide exclusively and exhaustively into true and false, arguments divide exclusively and exhaustively into valid and invalid, and argumentations divide exclusively and exhaustively into cogent and noncogent. In order for a proposition to be true it is necessary and sufficient for it to "correspond to fact". In order for an argument to be valid it is necessary and sufficient for its premise-set to logically imply its conclusion, or, what is the same thing, for the negation of the conclusion to be logically incompatible with the premises. According to Mates, the Stoics preferred to define validity in terms of incompatibility rather than implication. In order for an argumentation to be cogent it is necessary and sufficient for its chain of reasoning to make evident that its premise-set logically implies its conclusion. It is well worth the effort to note that propositions, arguments, and argumentations form three mutually exclusive ontological categories and that category mistakes ensue when a property appropriate to one category is attributed to something in another. It is incoherent to say that a proposition is valid or invalid, or cogent or noncogent. It is incoherent to say that an argument is true or false, or cogent or noncogent. Likewise it is incoherent to say that an argumentation is true or false, or valid or invalid. It goes without saying that the last three remarks apply only when the words are used in the senses here defined. It is also worth noting, as is rarely done, that the word proposition in English is often used in senses other than that being used here and that the word 'argument' is almost never used outside of logic in this sense. In fact there are many logicians who never use it in this sense, e.g. Lukasiewicz and Tarski.

The word deduction is used in English in several categories two of which are relevant to this discussion. In the first place, deduction is an epistemic process of extracting information that is implicit in given information. In logic this usage is sharpened up a bit and we say that deduction is the epistemic process of coming to know that a given single proposition is logically implicit in a given set of propositions. In other words, deduction amounts to the process by which we determine that a given argument is valid. In this sense of the word, 'deduction' is a process noun which does not pluralize. No occurrence of the pluralized word has this sense. Aristotle's theory of

deduction, to the extent that it was understood, dominated thinking on the subject until the 1920s and even then what became clear was that it needed supplementation, not that it needed correction.

In the second sense, a deduction is a result of an application of the process of deduction. In logic this sense is sharpened in various ways, and the variations can be important. Here a deduction is a cogent argumentation, an argumentation whose chain of reasoning makes evident that its conclusion is implied by its premise-set. Aristotle's theory of deduction is an account of how deductions are constructed or, what is the same thing given the context, how chains of reasoning are constructed.

According to Aristotle's theory there are certain simple valid arguments that can be seen to be valid without recourse to other valid arguments, in other words there are certain cases where the conclusion can be deduced from the premise-set immediately without the interposing of intermediate conclusions. Today inferences in such cases are called immediate inferences, where 'immediate' is taken in its etymological sense of "without intermediation" and not in any temporal sense.

Aristotle's theory of deduction recognizes two ways of coming to see that a given conclusion follows from given premises and accordingly he recognizes two types of deductions, the direct deductions and the indirect deduction. A direct deduction is obtained by chaining together immediate inferences in a sequence starting with given premises and ending with the conclusion. An indirect deduction is obtained by chaining together immediate inferences in a sequence starting with given premises augmented by the opposite of the conclusion and continuing until reaching a proposition whose opposite has already been reached.

The fact that Aristotle recognized the distinction between logical implication and logical deduction paralleling the distinction between truth and knowing attests to his penetrating analysis of proof. The fact that he divided a theory of deduction as part of his theory of proof is already sufficient to secure his reputation as the father of logic.

Bibliographic Note

The most recent articulation of the systematic epistemically-oriented interpretation of Aristotle's syllogistic is found in the Introduction to the Robin Smith translation of Prior Analytics (Hackett, Indianapolis, 1989). References below are cited from Smith's Bibliography. Corcoran 1972 and Smiley 1973 each give original and distinctive formulations of systematic epistemically-oriented interpretations; discussions, criticisms, modifications and refinements are found in several places including Clark 1980, Smith 1983, and Smith 1984. The ontically-oriented interpretation, which was developed by Lukasiewicz before World War II, is forcefully and clearly presented in Lukasiewicz 1957 and Patzig 1968.

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